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LETTER TO THE EDITOR

Transfer-matrix interface rescaling in magnetic bilayers

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Abstract. The transfer matrix method is applied to magnetic bilayer film with the aim of expressing its characteristic equation by the interface rescaling parameters. The interface-localized mode is then shown to exist in the system for appropriately chosen values of the interface spin pinning parameters defined in the model. We also show that in order to achieve a proper description of the interfacial conditions in the case of bilayer exchange coupled ferromagnetic films it is necessary to take into consideration a well defined 'effective interface pinning' originating in the intrinsic interfacial coupling.

In recent years studies of the magnetic properties of surfaces, multilayered structures and thin films have undergone a renaissance with the development of new growth and characterization techniques. Many new materials and structures have revealed significant new magnetic properties. Theoretical studies of such systems started but a few years ago. A direct numerical analysis of such systems would, in general, involve huge numerical computations. Therefore analytical approaches are of essential importance when studying such systems.

Several alternative analytical approaches to the analysis of composite structures have been made recently. The surface Green-function matching method has been used to obtain the eigenvalues of superlattices [1]. The interface response theory was formulated for determining eigenvalues [2] and eigenvectors [3] in composite systems. Another approach, the interface response rescaling method [4] permits the calculation of a response function of the composite system from the knowledge of the surface response functions of each constituent subsystem. Finally, the interface rescaling method [5] calculates the eigenvalues and eigenvectors of the finite composite layered system by performing the reduction of its eigenproblem to that of one of its individual constituent sublayers. This approach has recently been presented as a recursion interface rescaling procedure for solving the eigenproblem of double-layer [6] and triple-layer films [7]. This method has also been applied to the study of the properties of bilayer ferromagnetic films [8]. On the other hand quite another new approach [9–14] to the study of dynamical properties of magnetic multilayer films and superlattices has been made and this approach refers to the local picture of transfer matrix method. In the transfer matrix method, two parameters of the spin wave on one layer define the values of these two

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parameters on the next layer, and step by step the product of such transfer matrices defines these two parameters everywhere in the sample. Different choices of those two spin wave parameters have been used; from the early work by Poincaré in mechanics [15], the spin wave amplitude u_l and its 'gradient' $u_l - u_{l-1}$ have been used [11, 14], other authors used the amplitudes of two spin wave polarizations [12, 13]. Of course, these different transfer matrix approaches are equivalent since they are just due to a different choice of the basis of the vectorial set of spin waves. The transfer matrix approach is easy to handle numerically since it requires only products of 2×2 matrices even in the case of quite inhomogeneous materials. However, it requires many operations to find the eigenvalues of the characteristic equation, and this is the reason why more global methods were used when the accuracy of the numerical results was not so good for a conveniently short computational time. However, on the one hand computational times have decreased, and on the other hand the accuracy of the numerical definition of the eigenvalues can be easily checked by constructing the eigenvectors which are known to satisfy some simple analytical criteria.

Both methods, namely the interface recursion rescaling one and the transfer matrix method, consist in the successive elimination of the involved equations of motion—one by one, when approaching the interface. However, this elimination is performed by using different tricks and there is now a need to compare the final formulae obtained within the scopes of the two procedures. The aim of this paper is, firstly, to show that these two methods are fully equivalent. Additionally, it is worth noting that when performing the proof of this equivalence we have obtained as a by-product new formulae for the interface rescaling parameters expressed in terms of the transfer matrix elements. This is not a trivial result since now the rescaling interface parameters become directly explicit functions of the energy of the respective elementary excitations (e.g. magnons). Consequently, the characteristic equation can be expressed in terms of the energy as well, instead of the respective wave numbers, as it is in the standard procedure. This would become especially useful in the case of systems exhibiting strong bulk inhomogeneities, for example.

We consider the following eigenvalue problem:

$$\begin{aligned}
 (x - E/\alpha - a)u_0 - u_1 &= 0 & (l = 0) \\
 -u_{l-1} + (x - E/\alpha)u_l - u_{l+1} &= 0 & (l = 1, 2, \dots, N-2) \\
 -u_{N-2} + (x - E/\alpha - b)u_{N-1} &= \rho u_N & (l = N-1) \\
 \rho' u_{N-1} = (y - E/\beta - c)u_N - u_{N+1} & & (l = N) \\
 -u_{l-1} + (y - E/\beta)u_l - u_{l+1} &= 0 & (l = N+1, N+2, \dots, L-2) \\
 -u_{L-2} + (y - E/\beta - d)u_{L-1} &= 0 & (l = L-1)
 \end{aligned} \tag{1}$$

with the index l labelling the atomic planes ($l = 0; L - 1$ correspond to the surfaces, while $l = N - 1; N$ corresponds to the two interface planes). We set l such that the equations $l = 0, 1, \dots, N - 1$ (with $x - E/\alpha$ as diagonal term) describe the subsystem A, while the equations $l = N, N + 1, \dots, L - 1$ (with diagonal term $y - E/\beta$) describe the subsystem B. Above, E denotes an eigenvalue (the energy), u_l is the respective eigenfunction (the wavefunction) and α, β stand for bulk interaction parameters between nearest neighbours in the subsystems A and B respectively. ρ and ρ' are the interface coupling parameters, while the parameters a, d and b, c are, respectively, surface and interface pinning parameters.

We shall apply the transfer matrix procedure to the subset A only; the respective transformation for the subset B will be performed afterwards by rewriting the final result obtained for the subset A with recourse to the formal analogy existing between the two subsets. First, we note that any of the *bulk* equations of the subset A can be rewritten in terms of the transfer matrix **T** in the following manner [14]:

$$\begin{pmatrix} u_{l+1} \\ u_{l+1} - u_l \end{pmatrix} = \mathbf{T} \begin{pmatrix} u_l \\ u_l - u_{l-1} \end{pmatrix} \tag{2a}$$

with

$$\mathbf{T} = \begin{pmatrix} x - E/\alpha - 1; 1 \\ x - E/\alpha - 2; 1 \end{pmatrix}. \tag{2b}$$

If we now apply successively the transfer matrix **T** to each of the bulk equations of A, i.e. one after the other, we finally arrive at the following result:

$$\begin{pmatrix} u_{N-1} \\ u_{N-1} - u_{N-2} \end{pmatrix} = \mathbf{T}^{N-2} \begin{pmatrix} u_1 \\ u_1 - u_0 \end{pmatrix}. \tag{3}$$

Let us note, at this stage, that with regard to (3) one is justified in viewing the subset A of eigenvalue equations (1) as consisting now of three equations only, namely: the *surface* equation ($l = 0$), the *bulk* equation in matricial form, (3), and the *interface* equation ($l = N - 1$). By having recourse to the definition (2b) one easily finds the explicit expression for the $(N - 2)$ th power of the transfer matrix which appeared in (3). This is:

$$2q\mathbf{T}^{N-2} = \begin{pmatrix} T_{11}, T_{12} \\ T_{21}, T_{22} \end{pmatrix} \tag{4}$$

where

$$T_{11} = (p + q)(1 + p + q)^{N-2} - (p - q)(1 + p - q)^{N-2} \tag{5a}$$

$$T_{21} = (q^2 - p^2)[(1 + p + q)^{N-2} - (1 + p - q)^{N-2}] \tag{5b}$$

$$T_{12} = (1 + p + q)^{N-2} - (1 + p - q)^{N-2} \tag{5c}$$

$$T_{22} = (q - p)(1 + p + q)^{N-2} + (q + p)(1 + p - q)^{N-2} \tag{5d}$$

with the notations:

$$2p = x - E/\alpha - 2 \quad q = \sqrt{p(p + 2)}. \tag{6}$$

On the other hand, the surface and interface equations can be rewritten in the following forms:

$$u_1 - u_0 = (2p + 1 - a)u_0 \tag{7a}$$

$$u_{N-1} - u_{N-2} = \rho u_N - (2p + 1 - b)u_{N-1}. \tag{7b}$$

If we now substitute (7) and (4) into (3) we arrive at the final expressions for the functions u_{N-1} and u_N expressed by u_0 :

$$u_{N-1} = [(2p + 2 - a)(T_{11} + T_{12}) - T_{12}]u_0 \tag{8a}$$

$$\rho u_N = \{(2p + 2 - a)(T_{21} + T_{22}) + (2p + 1 - b)(T_{11} + T_{12})\} \\ - [T_{22} + T_{12}(2p + 1 - b)]u_0. \quad (8b)$$

Proceeding similarly we easily get the following expression for the eigenamplitudes at the remaining sites of the subsystem A:

$$u_l = u_0[(2p + 2 - a)(t_{11} + t_{12}) - t_{12}] \quad l = 1, 2, \dots, N - 2 \quad (8c)$$

where the newly introduced matrix elements t_{ij} are related to the matrix \mathbf{T} , equation (2b), by the equality: $t = \mathbf{T}^{l-1}$.

It is our aim to express the interface rescaling parameters (defined in [4]) by the transfer matrix elements T_{ij} derived in the preceding section. Using the notations of the present paper, the respective rescaling parameters are defined by the following equations:

$$\rho u_N = R^{B \rightarrow A} u_{N-1} \quad (9a)$$

and

$$\rho' u_{N-1} = R^{A \rightarrow B} u_N \quad (9b)$$

where $R^{B \rightarrow A}$ denotes the parameter rescaling the subset A with regard to the subset B, and $R^{A \rightarrow B}$ denotes the reverse. The concept of rescaling parameters as defined by (9) allows us to decouple the two respective subsets of (1). If use is made of one of the equations (9), decoupling of the two subsystems is effectively performed with the consequence that the respective rescaling parameter 'injects' the full information about one subsystem into the other. This is the main advantage of introducing the concept of the interface rescaling parameters which justifies our effort in searching for their explicit presentation in terms of the transfer matrix elements. By the definition (9b) and using the formerly obtained results (8a, b) we immediately find the formula for the rescaling parameter $R^{A \rightarrow B}$:

$$R^{A \rightarrow B} = \rho \rho' \frac{(2p + 2 - a)(T_{11} + T_{12}) - T_{12}}{(2p + 2 - a)[(T_{21} + T_{22}) + (2p + 1 - b)(T_{11} + T_{12})] - [T_{22} + T_{12}(2p + 1 - b)]}. \quad (10)$$

The formula for the remaining rescaling parameter $R^{B \rightarrow A}$ could in principle be obtained by re-applying the whole procedure presented above to the subsystem B. However, this would be an unnecessary task: on the grounds of the formal analogy existing between the subsets A and B of (1) we easily conclude that the formula for $R^{B \rightarrow A}$ is exactly the same as in (10), provided one replaces the quantities involved (see also (5) and (6)) in accordance with the following transformations: $N \rightarrow L - N$, $x \rightarrow y$, $\alpha \rightarrow \beta$, $a \rightarrow d$ and $b \rightarrow c$.

Now there is only one step to be made in order to arrive at the characteristic equation: on combining (9) we get equality:

$$\rho \rho' = R^{B \rightarrow A} R^{A \rightarrow B}. \quad (11)$$

From (10) and (5)–(6), the unknown quantity of (11) is the energy E (to power L). The roots of this equation are dependent both on the product of the interface coupling parameters ($\rho \rho'$) as well as on the pinning parameters involved (a , b , c and d) as one could expect from the formulation of the problem. After solving (11), knowledge of the eigenvalue set of E allows us to calculate the transfer matrix \mathbf{T} (using (2b)) for each of the normal excitations, whence the eigenfunctions u_1 can also be expressed in terms of E . This concludes our task.

To our knowledge, this is the first application of the transfer matrix approach to the bilayer eigenvalue problem aimed at the calculation of the respective interface rescaling parameters. The advantage of this approach consists in expressing the characteristic equation in terms of the energy rather than in terms of the respective wavenumber k , and this may prove useful in prospective applications of the method to systems which are non-homogeneous in the bulk, in which the k -dependence is very highly complex. In the case considered in the present paper, i.e. with subsystems exhibiting only surface inhomogeneities, the k -dependences are very simple, namely $x - E/\alpha = 2 \cos k_A$ and $y - E/\beta = 2 \cos k_B$. With these substitutions for the subsystem A we have (instead of (4) and (5)):

$$T^{N-2} = \frac{1}{\sin k_A} \begin{bmatrix} \sin(N-1)k_A - \sin(N-2)k_A; \sin(N-2)k_A \\ 2(\cos k_A - 1)\sin(N-2)k_A; \sin(N-2)k_A - \sin(N-3)k_A \end{bmatrix} \quad (12)$$

and for the respective transfer matrix of the subsystem B we need only to replace in the above formula k_A by k_B and N by $(L - N)$. When substituting appropriate matrix elements, resulting from (12), into (10), we obtain the following expression for the first rescaling parameter:

$$R^{A \rightarrow B} = \rho \rho' \frac{\sin Nk_A - a \sin(N-1)k_A}{\sin(N+1)k_A - (a+b) \sin Nk_A + ab \sin(N-1)k_A}. \quad (13)$$

Proceeding similarly we can get the respective expression for the second rescaling parameter. At this stage it already becomes obvious that the results obtained in [6] and expressed in the language of wavenumbers are therefore fully recovered.

We now consider a bilayer film composed of two (parallel-sided) layers made from the same ferromagnetic material characterized by the bulk exchange integral J_{bulk} . The last assumption leads, in our terminology and within the framework of the Heisenberg localized spin model, to the following equalities: $x = y$, $\alpha = \beta \equiv 1$. We are mainly interested in the spectrum of standing spin wave modes of our bilayer system, and the effects exerted upon it by the interface spin pinning. To emphasize this interface effect we neglect in our present considerations the surface anisotropy, i.e. we assume that $a = d = 1$. However, we assume throughout that the magnetizations of the interacting sublayers are parallel to each other. We perform calculations within the framework of the Heisenberg localized spin model assuming pure exchange (nearest-neighbour) interactions and a Zeeman Hamiltonian in standard forms. We neglect the dipolar interactions since we are mainly interested in the range of energies corresponding to pure exchange spin wave excitations at low temperatures, which can be investigated, e.g. by the spin wave resonance experiment. Let us consider first the case when $J_{\text{interface}} \approx J_{\text{bulk}}$. Introducing now the interface anisotropy field K_{int} acting on interface spins only (i.e. on those belonging to the planes $l = N - 1; N$)—thus being the field responsible for interface spin pinning—we arrive at the following expression for the interface pinning and coupling parameters [8]:

$$b = c = 1 - \frac{g\mu_B}{2SJ_{\text{bulk}} z_{\perp}} (\boldsymbol{\gamma} \cdot \mathbf{K}_{\text{int}}) \quad \rho = \rho' = 1 \quad (14)$$

where z_{\perp} is the number of nearest spins situated in the sub-interface plane, $\boldsymbol{\gamma}$ is the film magnetization unit vector and other notations have the standard meaning. From (14) we notice that interface spin pinning depends both on the interface anisotropy field

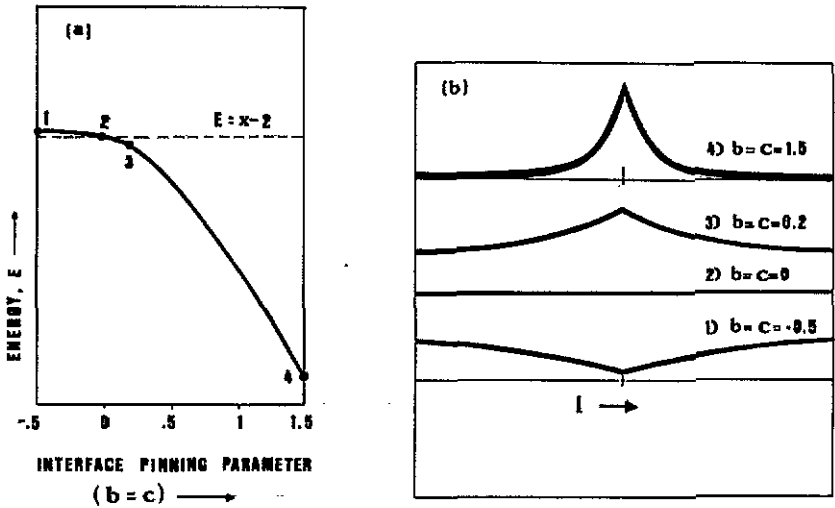


Figure 1. Energy and shape of the first ($n = 1$) energetically lowest lying spin wave mode in the spectrum versus the interface pinning. For simplicity it is assumed here that the bilayer film consists of two identical sub-layers of equal thicknesses ($L = 2N$) with equal interface coupling parameters: $\rho = \rho' = 1$. It is also assumed that there are no surface anisotropies (i.e. $a = d = 1$) on the outer surfaces. (a) shows the dependence of the mode energy on the interface pinning parameter ($b = c$), while (b) shows the evolution of the corresponding mode profile. When the interface pinning parameter grows, the respective interface spin pinning becomes weaker, and this results in changing the mode character from 'bulk' (case 1), through uniform mode (case 2), to interface-localized mode (cases 3 and 4).

value, as well as on the angle between this anisotropy field and the film magnetization. Figure 1 shows the effect exerted by the interface pinning on the energetically lowest ($n = 1$) lying mode of the spin wave spectrum; one sees that for $b < 0$ the energy (figure 1(a)) of this mode lies *within* the 'band' of extended (bulk) modes. However, for growing positive b it is shifted below the band edge (denoted as a broken line) to the 'forbidden' zone. From figure 1(b) it is evident that the mode belonging to the forbidden zone has the nature of an interface-localized mode, whose localization increases when b grows. Physically, as follows from the formula (14), such interface localization can be achieved for the appropriate direction of the film magnetization with respect to the interface anisotropy field.

Let us now consider the opposite case, namely, the case when $K_{\text{int}} \equiv 0$ but $J_{\text{interface}} \neq J_{\text{bulk}}$. The respective interface pinning and coupling parameters become (see [8] and [16]):

$$b = c = 1 - J_{\text{interface}}/J_{\text{bulk}} \quad (15a)$$

$$\rho = \rho' = J_{\text{interface}}/J_{\text{bulk}}. \quad (15b)$$

The exchange integral $J_{\text{interface}}$ represents *intrinsic* interfacial coupling, whereas the parameters ρ, ρ' can be dealt with as *effective* interfacial coupling parameters. Similarly, the parameters b, c can be termed the effective interfacial *pinning* parameters; we note that they are functions of the intrinsic interfacial coupling $J_{\text{interface}}$. The *effective* coupling and pinning parameters are mutually interdependent: a weakening in effective coupling at decreasing $J_{\text{interface}}$ is compensated for by an increase in effective pinning (see (15)).

Effective interfacial pinning can easily be understood on grounds of the molecular fields acting through the interface—we find that the effective interface pinning parameters b , c (equation (15a)) can be re-expressed as follows:

$$b = c = 1 - (g\mu_B/2SJ_{\text{bulk}} z_{\perp})(\gamma \cdot \mathbf{K}_{\text{int}}) \quad (16)$$

where

$$\mathbf{K}_{\text{int}} = (2z_{\perp} J_{\text{interface}}/g\mu_B)\mathbf{S} \quad (17)$$

is effective interfacial molecular field acting on the interface spins \mathbf{S} (of a given sublayer) originating in the interactions of these spins with the spins of the adjacent sublayer.

The intrinsic antiferromagnetic interfacial coupling can also be responsible for the emergence of interface localized spin wave modes as can (as we have seen above) the interface anisotropy field. However, we expect that the effect of such localization should be relatively enhanced in the present case, since $J_{\text{interface}}$ acts simultaneously *both* through the interface coupling parameters as well as through the interface pinning parameters (as one sees from (15)). At this stage it is not our aim to proceed to a numerical analysis of this effect and we leave the problem for further investigation. In conclusion, however, let us emphasize that we believe our method will also turn out to be suitable for studying interface localized modes (which have recently attracted the interest of experimentalists) in bilayer films consisting of two different ferromagnetic materials.

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